Security Price Informativeness with Delegated Traders

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Abstract

Trade in securities markets is conducted by agents acting for principals, using “mark-to-market” contracts whereby performance is assessed using security market prices. We endogenize contract choices, information production, informed trading, and security price informativeness. But there is a contract externality. Prices are informative only because other principals induce their agents to trade based on privately produced information. The agent-traders then have an incentive to coordinate and shirk. The market price is less informative, reducing the effectiveness of mark-to-market contracts. By using managerial discretion to vary the contract type unpredictably, principals mitigate traders’ coordinated manipulation and improve price informativeness. (JEL G12, G14)

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An important step in the intellectual foundation of efficient markets was the solution to the problem of how information comes to be embedded into security prices. Sanford Grossman and Joseph Stiglitz (1980) addressed this issue by introducing the notion of “noise” - a noisy supply curve for the asset, in their case. This allowed some traders to produce costly information and trade profitably on their findings, with the information being impounded in the price (see James Dow and Gary Gorton (2008) for a review). In this paper, we revisit the issue of the informativeness of security prices in the context of delegated trading, the more realistic setting. We show that there are several other sources of noise that arise from contract externalities when trading is delegated. These other sources of noise also endogenously determine the informativeness of security prices.

In the context of a security market, some traders or portfolio managers are hired to produce and trade on private information on behalf of investing principals, who must design compensation contracts to control the traders’ trading behavior. In contrast to the standard agency paradigm, in which an agent’s contractual compensation is only a function of the final value or price, a mark-to-market contract involves interim price information from security markets, information that arrives between the contract initiation date and the final payoff. In the optimal mark-to-market contract, the trader gets a high payoff if his trading position is consistent with the interim price and the interim price is informative about the fundamentals, or, if he delivers a high final return when the interim price is not informative about the fundamentals. When market prices are used to monitor the behavior of agent-traders, each principal relies on the other principals to induce their agent-traders to gather information so as to make prices informative. But, this externality can lead the agent-traders to coordinate joint shirking, a kind of manipulation of prices, resulting in less informative prices and ultimately making mark-to-market contracts undesirable.

The possible manipulation of security prices induced by agent-traders’ compensation contracts generates an incentive for principals to choose the “standard contract.” With a standard contract a trader’s compensation is not contingent on interim market price information, but rather based solely on the final realization of the portfolio return. A standard contract ignores valuable information contained in the interim price, but it prevents manipulation based on joint shirking. However, adopting the standard contract means that the price will be, at least weakly, informative, creating an incentive for other principals to free ride on this information using mark-to-market contracts for their agent-traders. When the propensity of shirking is high, the equilibrium then must be one in which the principals follow mixed strategies in contracts, sometimes adopting mark-to-market contracts and sometimes adopting standard contracts. This introduction of noise, interpreted as randomness in principals’ managerial discretion over compensation, is realistic and makes it harder for the agent-traders to manipulate their payoffs. This equilibrium contract randomization mitigates the contract externalities because it reduces the dependence of the agent-traders’ payoff on market price.

1 In the managerial contracting literature, it has been demonstrated that the firm’s market price should be included in the optimal contract for the manager as long as it contains information not reflected by the firm’s current or future profit data; see, for example, Bengt Holmstrom and Jean Tirole (1993).
Our paper contributes to the strand of the literature that examines the impact of agency problems on asset pricing. For example, Franklin Allen and Gorton (1993) show that when there is asymmetric information between investors and portfolio managers, portfolio managers have an incentive to churn, which can result in price bubbles. Dow and Gorton (1997) study a model in which noise trade by some managers causes high levels of turnover. Our paper incorporates several new elements, including mark-to-market contracts, agents’ coordinated manipulation and principals’ contract choices.

We study a game with multiple principal-agent pairs, which is related but different from models with multiple principals competing for multiple common agents (for example, see Michael Peters (2001) and Andrea Prat and Aldo Rustichini (2003)). The competition between principals in choosing contracts for their competing agents that we analyze is similar to some previous literature in industrial organization. Steven D. Sklivas (1987) and Chaim Fershtman and Kenneth J. Judd (1987) study this type of contracting problem in a duopoly setting with Cournot or Bertrand competition. In a similar financial market setting, Alexander Gümbel (2005) studies the asset allocation problem in a multiple principal-agent environment, but the contract is restricted to be linear in relative performance. We study mark-to-market contracts and the induced manipulation, a problem that arises in models with a single principal and multiple common agents (see Jean-Jacques Laffont and David Martimort (1997, 2000) and Yeon-Koo Che and Jinwoo Kim (2006)). In our model, price informativeness is jointly determined by principals’ contract choices and agents’ coordinated manipulation.

The agency relation we study is pervasive in security and derivatives trading. Professional traders work at hedge funds, mutual funds, money management firms, and banks. In over-the-counter markets, like those for government securities, foreign currency, corporate bonds, residential and commercial mortgage-backed securities, other asset-backed securities, and derivative securities (comprising interest rate, equity, foreign currency, credit, commodity, energy and other derivatives), the entire market is based on traders working for others. These are important markets. For example, the notional amount of interest rate derivatives in June 2007 was $346.9 trillion and the amount of credit default swaps outstanding in June 2007 was $42.6 trillion (see Bank for International Settlements (2007)). Since delegated traders in these markets invest with

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2 There is a large literature on delegated portfolio management, which focuses on issues of the manager's compensation structure. For example, Sudipto Bhattacharya and Paul Pfleiderer (1985), Michael J. Brennan (1993), Anat R. Admati and Pfleiderer (1997), Domenico Cuoco and Ron Kaniel (2000), and Hui Ou-Yang (2003). For the most part, the issue in these studies concerns the choice of a “benchmark” to evaluate the manager's performance.

3 Guillaume Plantin, Haresh Sapra, and Hyun Song Shin (2008) and Allen and Elena Carletti (2008) study the effects of mark-to-market regime, but they do not study endogenous contract choices and price informativeness.

4 Even in the public equity market, most trading is delegated to professional portfolio managers. In 2004, U.S. households directly held less than 40% of corporate equities, while they held about 90% in 1950 and 70% in 1970 (see “Flow of Funds” issued by the Federal Reserve Board). Also according to the survey results released by the Investment Company Institute (ICI) and the Securities Industry Association (SIA), in 2002, 89% of the investors invested in mutual funds and 58% of the investors relied on professional financial advisers when making investment decisions.
other people’s money, principal investors want to make sure that traders invest based on superior information rather than blind gambling. Interim market price is often used as a benchmark to check traders’ trading behavior. The mark-to-market contract we study resembles the real world restrictions imposed on traders for this purpose. For example, some mutual funds prohibit traders from trading stocks whose prices have fallen below a certain level, such as sub-$5 stocks; other funds impose short sales restrictions on traders; and there are also funds adopting strict loss-cutting policies. The rationale behind these policies is to control risk. When traders deviate too much from the market, suspicion of risky speculation arises. The mark-to-market contract we study reflects this sort of risk control. Interestingly, financial institutions in the same industry offer contracts with different compensation structures or different restrictions. We provide a reason for this variation. Papers that study the disciplining effects of interim mark-to-market returns on delegated portfolio management include Judith Chevalier and Glenn Ellison (1997), Stavros Panageas and Mark Westerfield (2009), and Kaniel and Julien N. Hugonnier (2010). These papers focus on risk-seeking behavior induced by an option-like compensation scheme, while we focus on coordinated random gambling (trading without information) induced by a marked-to-market payoff structure.

The paper consists of two parts. In the first part we set up a model that incorporates the agency problem in a market microstructure model. We demonstrate how market information can help reduce agency costs. Section I introduces the model and states some assumptions. In Section II, we solve for the optimal standard contract. It is the benchmark case, yet a suboptimal solution. We then solve for the optimal mark-to-market contract and compare it with the standard contract.

In Section III we consider the case of many principal-agent pairs. The interaction of the agency problem and the contract externality give rise to market manipulation. The externality caused by the feedback from traders’ trading activity makes it a nontrivial contract choice for principal-investors at the initial date. When the manipulation propensity is large enough, principals use contract randomization to mitigate the agency problem, resulting in additional noise in the asset price.

We conclude in Section IV.

I. Model Setup

Our focus is on an agency problem in which a principal-investor hires an agent-trader to trade on his behalf in a security market. Like the standard agency problem, the principal must design a contract to induce the agent-trader to make an effort to improve the value of the realization of the “project,” in this case a trading position. In our setting the principal observes an interim price signal which can be used to check the agent-trader’s behavior. Thus, the strategy space of the agent-trader is larger than in the standard problem, introducing the issue of “risk management” in a way which is not present in the usual problem. The risk management issue concerns how the principal can control the agent-trader’s hidden action using his observable actions in combination with market
prices in the interim securities market. We first investigate this in the simplest setting with one principal-agent pair and later extend to the case with multiple principal-agent pairs. We will call the agent-trader the “professional trader” or simply the “trader,” as he is hired by the principal.

The security market has five types of risk neutral participants: a direct investor, an indirect investor, a professional trader, liquidity (or noise) traders, and a market maker. The indirect investor (the principal) has the money to invest, but no access to the technology to become informed, so he hires the trader (the agent) to make the investment. The direct investor invests for himself; he has no need to hire an agent-trader. In Section III, we will replace the direct trader with another principal-agent pair. Both the professional trader and the direct investor have the technology to become imperfectly informed about the value of the security at a final date. However, the trader has an incentive to shirk. If he chooses not to make an effort to acquire information, he receives a shirking benefit of \( \kappa > 0 \). For ease of exposition, we will refer to the security as “stock.”

There are three dates in this economy:

- **Date 0**: The trader and the principal sign a contract which authorizes the trader to trade on behalf of the principal and specifies the payoffs to the trader under each contingency observable to both parties to the contract.

- **Date 1**: Both the direct investor and the trader (if he does not shirk) receive a signal about the terminal value of the stock. Then each decides whether to submit a buy order or a sell order. For simplicity, investors can trade a maximum of \( x \) shares. The liquidity traders trade an amount \( \delta \sim N(0, \sigma^2) \). The market maker sets the price based on the total order flow by forming the conditional expectation of the security value, given knowledge of the model, as is standard in financial economics following Kyle (1985).

- **Date 2**: The liquidation value of the stock is realized, and the principal pays his trader according to the contract.

The liquidation value of the stock has the following distribution:

\[
\begin{align*}
  v &= \begin{cases} 
    v_H & \text{with probability } 1/2 \\
    v_L & \text{with probability } 1/2,
  \end{cases} \\
  \end{align*}
\]

(1)

and we assume \( v_H > v_L \).

For simplicity, the direct investor and the trader (if he does not shirk) receive the same

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5 The existence of a direct investor makes the price informative even when the agent trader shirks (that is, does not produce information) and simply trades randomly. “Liquidity traders” or “noise traders” trade for exogenous reasons and, on average, lose money, allowing other agents to produce costly information and still make a profit. See Dow and Gorton (2008).

6 The assumption of equal probability for \( v_H \) and \( v_L \) is not crucial for our results.
private signal, which can be either \( s_H \) or \( s_L \), and the signal is correlated with the true liquidation value of the stock. The probability distribution of \( v \) conditional on the signal \( s \) is:

\[
\begin{align*}
\text{prob}(v | s_H) &= \text{prob}(v | s_L) = \theta \\
\text{prob}(v | s_H) &= \text{prob}(v | s_L) = 1 - \theta,
\end{align*}
\]

and we assume \( \theta > 1/2 \).

When both the direct investor and the agent-trader trade on privately produced information, the distribution of the market order flow \( z \) is a mixture of two Normal distributions:

\[
z \sim \frac{1}{2} \mathcal{N}(2x, \sigma^2) + \frac{1}{2} \mathcal{N}(-2x, \sigma^2). \tag{3}
\]

Let \( \phi^+(z) \) denote the probability density function for the distribution \( \mathcal{N}(2x, \sigma^2) \), \( \phi^-(z) \) denote the probability density function for the distribution \( \mathcal{N}(-2x, \sigma^2) \), and \( \phi^0(z) \) denote the probability density function for the distribution \( \mathcal{N}(0, \sigma^2) \).

It is easy to show that \( \phi^+(z)/\phi^0(z) \) is increasing in \( z \) and \( \phi^-(z)/\phi^0(z) \) is decreasing in \( z \). The monotonicity of the likelihood ratio \( \phi^+(z)/\phi^0(z) \) tells us that the higher the order flow \( z \), the more likely it is from the distribution \( \mathcal{N}(2x, \sigma^2) \). A similar argument applies for the likelihood ratio \( \phi^-(z)/\phi^0(z) \). This property of the monotone likelihood ratio is important for characterizing the optimal contract.

Let \( p_H \) denote the expected liquidation value at date 3 conditional on the signal \( s_H \) being received at date 2 by the trader and the direct investor; \( p_L \) is defined similarly conditional on the signal \( s_L \) being received. Then:

\[
\begin{align*}
p_H &= E[v | s_H] = \theta v_H + (1 - \theta)v_L \\
p_L &= E[v | s_L] = \theta v_L + (1 - \theta)v_H.
\end{align*}
\]

The market maker cannot observe the contract or the identity of traders, but he rationally expects the traders’ trading behavior and sets the price to be the expected liquidation value conditional on the total order flow; see Kyle (1985). Therefore, if both the direct investor and the agent-trader trade truthfully based on the produced information, the price can be expressed as a function of the order flow \( z \) as follows:

\[
p(z) = \frac{\phi^+(z) p_H + \phi^-(z) p_L}{\phi^+(z) + \phi^-(z)}. \tag{5}
\]

We can show \( dp(z)/dz > 0 \), \( \lim_{z \to \infty} p(z) = p_H \), and \( \lim_{z \to -\infty} p(z) = p_L \). From the
magnitude of the total order flow, the market maker infers the news received by the informed agents. The larger the order size, the more likely it is that the good news was received, and thus the market maker sets a higher price. Similarly when the market maker receives a large sell order, he is pretty sure that bad news was received and sets a lower price.

In Section II below, we will study two types of contracts. The first type of contract only depends on the final value or price, not on interim information. This is the typical contract that is studied in the standard agency paradigm and we call it the standard contract. Since it does not employ all information available in the market, it is suboptimal. The second type of contract stipulates that the payoff to the agent depends on his trading position, \( \lambda \), the final liquidation value of the security, \( v \), and the interim market price, \( p \). We call it the mark-to-market contract. Since a trading position, \( x \) or \(-x\) is equivalent to the trader claiming to have received a good or bad signal, there is no need to reward the trader for not fully utilizing the information. In other words, we can assume that the trader receives a zero payoff for a trading position other than \( x \) or \(-x\), and only consider a trading position of \( x \) (buy) or \(-x\) (sell).

II. The Monitoring Cost Saving of a Mark-to-Market Contract

A. The Standard Contract

As a benchmark case, we first solve for the optimal standard contract before we study contracts that are marked-to-market.

ASSUMPTION 1: There is an upper bound \( \bar{w} \) for the wage payment to the agent-trader.

The role of this assumption is to guarantee a bounded optimal solution. This will become clear as we proceed; the interpretation of \( \bar{w} \) will also be discussed later.

We first characterize the optimal standard contract. Define \( W^S \) as follows:

\[
W^S \equiv \{ w^S (\lambda, v) \mid 0 \leq w^S (\lambda, v) \leq \bar{w} \text{ for any } \lambda \text{ and } v \}. \quad (6)
\]

\( W^S \) is the set of all feasible standard non-mark-to-market contracts.

Let \( w_{\eta} \) be the wage payment to the trader when his trading position is \( \lambda \) (buy or sell \( x \) shares), and the realized liquidation value is \( v_\eta \). A standard contract \( w^S(\lambda, v) \), characterized by four numbers \( \{ w_{bH}, w_{bL}, w_{sH}, w_{sL} \} \), is incentive-compatible if it satisfies:

\[\text{We assume that contracts cannot be signed contingent on other traders’ security positions. This information is not observable or verifiable.}\]
\begin{align}
\text{(IC.S1)} & \quad \partial w_{bH} + (1 - \theta) w_{bl} \geq \partial w_{sH} + (1 - \theta) w_{sl} \\
& \quad (1 - \theta) w_{sH} + \partial w_{sl} \geq (1 - \theta) w_{bH} + \partial \kappa_{bl},
\end{align} 

and

\begin{align}
\text{(IC.S2)} & \quad \frac{1}{2} [\partial w_{bH} + (1 - \theta) w_{bl} + (1 - \theta) w_{sH} + \partial w_{sl}] \\
& \quad \geq \max \left\{ \frac{1}{2} (w_{bH} + w_{sl}) + \kappa, \frac{1}{2} (w_{sH} + w_{sl}) + \kappa \right\},
\end{align} 

(IC.S2) can be rewritten as:

\begin{align}
\text{(IC.S2)} & \quad \frac{1}{2} \left[ (1 - \theta) w_{sH} + \partial w_{sl} \right] \geq \frac{1}{2} \left[ \partial w_{bH} + (1 - \theta) w_{bl} + \kappa \right] \\
& \quad \frac{1}{2} \left[ (1 - \theta) w_{sH} + \partial w_{sl} \right] \geq \frac{1}{2} \left[ (1 - \theta) w_{bH} + \partial w_{bl} \right] + \kappa.
\end{align} 

and it is clear that (IC.S2) implies (IC.S1).

(IC.S1) says that, when the trader receives a good signal, he is better off submitting a buy order, and when he receives a bad signal, he is better off submitting a sell order. (IC.S2) says that the trader is better off by acquiring information and trading in the best interests of the principal instead of shirking (getting $\kappa$) and randomly buying or selling the stock.

The expected payoff to the trader can be written as:

\[ A^S = \frac{1}{2} [\partial w_{bH} + (1 - \theta) w_{sH} + \partial w_{sl} + (1 - \theta) w_{bl}] . \]  

The optimal standard contract $w^S(\lambda, v)$ solves the following programming problem:

\[ \min_{w^S \in W^S} A^S \text{ subject to the IC conditions in (9).} \]

The constraints in (9) have to be binding; otherwise the principal investor could always lower the trader’s compensation without violating the IC conditions. The following proposition states that the optimal compensation contract is simple: the trader will either get a positive payoff or zero, depending on how his position aligns with the final liquidation value.

**PROPOSITION 1 (Optimal Standard Contract):** The optimal standard contract $w^S(\lambda, v) = \{w_{bH}, w_{bl}, w_{sH}, w_{sl}\}$ solving (11) takes the following form:

\[ w_{bH} = w_{sl} = \frac{2\kappa}{2\theta - 1}, \text{ and } w_{sH} = w_{bl} = 0. \]
PROOF: See Appendix. ■

The standard contract stipulates a pure performance-based compensation scheme: the trader gets a high payoff only when the final return is positive.

When all informed participants trade truthfully, the expected payoff derived from informed trading is:

\[
V = \frac{1}{2} x \left[ \int_{-\infty}^{\infty} (p_H - p(z))\phi^+(z)dz + \int_{-\infty}^{\infty} (p(z) - p_L)\phi^-(z)dz \right] 
\]

\[
= x(p_H - p_L)\int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz. \tag{13}
\]

The expected wage payment to the agent trader is:

\[
A^s = \frac{2\theta\kappa}{2\theta - 1}. \tag{14}
\]

In order for a principal investor to hire an agent-trader with a standard contract, two conditions have to be satisfied. First, the principal investor needs to reward the trader enough to induce information production. In other words,

\[
\bar{w} \geq \frac{2\kappa}{2\theta - 1}. \tag{15}
\]

Second, the principal investor’s participation constraint has to be satisfied:

\[
V - A^s \geq 0 \quad \Rightarrow \quad x(p_H - p_L)\int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz \geq \frac{2\theta\kappa}{2\theta - 1}. \tag{16}
\]

Henceforth we make the following assumption:

ASSUMPTION 2: \( x(p_H - p_L)\int_{-\infty}^{\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz \geq (1 + \frac{1}{2\theta - 1})\kappa \) and \( \bar{w} \geq \frac{2\kappa}{2\theta - 1} \).

The above assumption speaks to the two roles that a small \( \kappa \) plays: (i) A small \( \kappa \) ensures that the incentive-compatible compensation is lower than the gross information gain such that a principal is willing to hire an agent trader; and (ii) A small \( \kappa \) ensures that the incentive-compatible compensation is lower than the value of \( \bar{w} \), the maximum the principal can pay. The agent-trader’s expected compensation is equal to \( 2\theta\kappa/(2\theta - 1) \), which is greater than the cost he incurs to collect information, \( \kappa \). The difference is \( \kappa/(2\theta - 1) \), which can be interpreted as the agency cost due to the moral hazard problem.

Since the standard contract does not incorporate market information into monitoring, it is
suboptimal relative to a mark-to-market contract, which uses the information contained in the interim price and reduces the monitoring cost. We will discuss this in more detail in the next subsection.

B. The Mark-to-Market Contract

To simplify the notation, we define an agent-trader’s expected payoff as follows:

\[ A_{ij}^x = \int_{-\infty}^{\infty} w(p(z), \lambda, v, \eta)\phi^x(z)dz, \quad (17) \]

where \( \chi = +, -, 0; \lambda = b \) or \( s \); and \( \eta = H \) or \( L \). \( A_{ij}^x \) is the expected payoff to the trader when his trading position is \( \lambda \) (buy or sell \( x \) shares), the informed order flow is \( \chi \), and the realized liquidation value is \( v, \eta \).

Similar to a standard contract, a mark-to-market contract \( w(p, \lambda, v) \) is incentive-compatible if it satisfies:

\[ \theta A_{bhH} + (1 - \theta) A_{bhL} \geq \theta A_{bH}^0 + (1 - \theta) A_{bL}^0 \]
\[ (1 - \theta) A_{bhH} - \theta A_{bL} \geq (1 - \theta) A_{bH}^0 + \theta A_{bL}^0, \quad (18) \]

and

\[ \frac{1}{2} \left[ \theta A_{bhH} + (1 - \theta) A_{bhL} \right] \geq \frac{1}{2} \left[ \theta A_{bH}^0 + (1 - \theta) A_{bL}^0 \right] + \kappa \]
\[ \frac{1}{2} \left[ (1 - \theta) A_{bhH} - \theta A_{bL} \right] \geq \frac{1}{2} \left[ (1 - \theta) A_{bH}^0 + \theta A_{bL}^0 \right] + \kappa. \quad (19) \]

It is clear that (IC.2) implies (IC.1). Intuitively, if a trader collects information at a cost ex-ante, then he will make a full use of it ex-post.

The expected payoff to the trader can be written as:

\[ A = \frac{1}{2} \left[ \theta A_{bhH} + (1 - \theta) A_{bhL} + (1 - \theta) A_{bhH} + \theta A_{bhL} \right] \quad (20) \]

and the optimal contract \( w^*(p, \lambda, v) \) solves the following programming problem:

\[ \text{Min}_{w \in W} A \text{ subject to the IC conditions in (19).} \quad (21) \]

Before we proceed to solve for the optimal contract \( w^*(p, \lambda, v) \), we claim that the constraints in (19) have to be binding; otherwise we could lower either \( w(p, b, v) \) or \( w(p, s, v) \) to improve the contract.

The following proposition shows that the optimal compensation contract is simple; the
trader will either receive a positive fixed compensation, \( w \), or zero, depending on how his trading position (\( \lambda \)) aligns with the interim price (\( p \)) and the final liquidation value (\( v \)).

**Proposition 2 (The Optimal Mark-to-Market Contract):** Given the pricing function, (5), the optimal contract \( w^*(p, \lambda, v) \) solving (21) is characterized by four cutoff values, denoted as \( z^* = \{ z_{bH}^*, z_{bL}^*, z_{sH}^*, z_{sL}^* \} \) such that: (i) for any \( p < p(z_{bH}), w^*(p, b, v_H) = 0 \), and for any \( p \geq p(z_{bH}), w^*(p, b, v_H) = \bar{w} \); for any \( p > p(z_{sH}), w^*(p, s, v_H) = 0 \), and for any \( p \leq p(z_{sH}), w^*(p, s, v_H) = \bar{w} \). (ii) \( z_{iH}^* \) satisfy the following conditions:

\[
\frac{1}{2} \bar{w} \left[ \mu(1 - \Phi^*(z_{bH}^*)) + (1 - \theta)[1 - \Phi^*(z_{bL}^*)] \right] = \frac{1}{2} \bar{w} \left[ \theta \Phi^0(z_{sH}^*) + (1 - \theta) \Phi^0(z_{sL}^*) + \kappa \right],
\]

\[
\frac{1}{2} \bar{w} \left[ (1 - \theta) \Phi^-(z_{sH}^*) + \theta \Phi^-(z_{sL}^*) \right] = \frac{1}{2} \bar{w} \left[ (1 - \theta)[1 - \Phi^0(z_{bH}^*)] + \theta[1 - \Phi^0(z_{bL}^*)] \right] + \kappa,
\]

and

\[
\frac{1}{2} \bar{w} \left[ \mu(1 - \Phi^*(z_{bH}^*)) + (1 - \theta)[1 - \Phi^*(z_{bL}^*)] \right] = \frac{1}{2} \bar{w} \left[ (1 - \theta)[1 - \Phi^0(z_{bH}^*)] + \theta[1 - \Phi^0(z_{bL}^*)] \right] + \kappa,
\]

where \( \Phi^*(\cdot) \) is the cumulative distribution functions corresponding to the probability distribution functions \( \phi^*(\cdot) \).

**Proof:** See Appendix. ■

Proposition 2 says that the optimal contract is marked-to-market, that is, it depends on the interim security market price and the final liquidation value, as well as the interim trading position of the trader (buy or sell). Because the direct investor always trades truthfully, there is information in the interim price. This information can be used by the principal to provide the trader with an incentive to acquire information and trade accordingly. The optimal contract punishes the trader when both the interim market price and the final liquidation value are against the direction of his trade (buy or sell) at the interim date.

The four equations in Proposition 2, equations (22) – (23), have four unknowns, \( z_{bH}^*, z_{bL}^*, z_{sH}^*, z_{sL}^* \), which characterize the optimal contract. As the pricing function in (5) is monotone in \( z \), it is perhaps easiest to think of the cutoff values in \( z \) as corresponding to prices. Then, the proposition says that the optimal contract consists of a step function (this is the result of monotone likelihood ratio property of the normal distribution, as discussed earlier) in terms of the security price \( p \) for each pair of \( \lambda \) and \( v \).

To interpret the optimal contract, divide it into three parts: (i) the first part rewards consistency with the market price; (ii) the second part punishes deviations from the market price; and (iii) finally, there is the return-based compensation. For example, when the trader has a long position (he bought the stock), part (i) corresponds to the case when
\( p > p(z_{hl}^*) \); part (ii) is the case when \( p < p(z_{sh}^*) \); and part (iii) is the case when \( p \) is in between. It is easy to see that part (iii) is the usual return-based contract for the trader; a higher return generates a higher payoff for the trader. Parts (i) and (ii) are new. With small probability, the trader could get a positive wage when the return is negative and gets nothing when the return is positive, but this only happens when the price is informative about the fundamentals. Due to a positive correlation between traders’ signals (here they are perfectly correlated), a trader is rewarded if his position is consistent with the market price movement, which is linked to the other trader’s position resulting from the information he produced.

In summary, the principal wants to compensate a trader who trades on information instead of trading randomly. The principal imposes the requirement of consistency between the trader’s position and the market price. If all traders in the market make the same mistake, they are punished; if a trader is the only one taking some position, he is punished as it is likely he is speculating. It may seem unreasonable not to reward a trader when he makes a lot of money. However, from the perspective of risk management, it is important to make sure that traders do not gamble. On the other hand, as we will discuss later, it is this “consistency-reward” mechanism that leaves room for traders to manipulate the market without producing information.

In terms of the trader’s expected compensation, Proposition 2 implies that \( A_{bh}^* > A_{bl}^* \) and \( A_{sh}^* > A_{sl}^* \), direct results of (22). So the trader receives a lower payoff if the realized liquidation value contradicts the trader’s prior trading decision, on average. In other words, he gets a lower payoff if he bought the stock, but its final value is \( v_L \). Also, notice that the difference between \( z_{bh}^* \) and \( z_{bl}^* \) (or \( z_{sh}^* \) and \( z_{sl}^* \)) is greater when the signal is more accurate, that is, when the value of \( \theta \) is greater. Intuitively, the trader gets a harsher punishment if the probability of receiving an incorrect signal is lower.

Although the closed-form optimal mark-to-market contract is derived under specific parametric assumptions, the two aspects of the results should be valid for more general setups. First, the interim market price provides additional information to tighten the compensation contract. Second, the mark-to-market contract checks the alignment between the trader’s trading position with two variables: the interim price and the final liquidation value. When the interim price is not very informative about the fundamentals (that is, the price is in the interim range), the alignment with the final liquidation value dominates, similar to usual performance contracts. When the interim price is very informative about the fundamentals (that is, the price is extremely high or extremely low), the alignment with the interim price dominates and it guarantees that the trader does not engage in risky speculation.

**PROPOSITION 3 (Existence and Uniqueness of the Optimal Mark-to-Market Contract):** Under Assumptions 1 and 2, there exists a unique optimal mark-to-market contract.\(^8\)

\(^8\)The existence of an optimal mark-to-market contract requires a weaker condition than Assumption 2, which also guarantees the incentive compatibility and the principal’s profitability of a standard contract.
PROOF: See Appendix. ■

The standard contract is a feasible choice because it also satisfies (IC.2) in (19), but it is dominated by the mark-to-market contract because the latter tightens monitoring through using the market price, which is informative. The gross payoffs from the informed trading are exactly the same no matter whether the contract is standard or marketo-market, however, the monitoring cost is smaller with the mark-to-market contract.

The expected payoff of the investment under informed trading is the same as \( V \) defined in (13), which is affected by \( \sigma \) and \( \theta \), but not affected by \( \kappa \) or \( \overline{w} \). However, \( \kappa \) or \( \overline{w} \) affect the contractual payoff distribution between the principal and the trader. We summarize the payoff dependency in the proposition below.

PROPOSITION 4 (Comparative Statics): Under the optimal mark-to-market contract, (i) the joint expected payoff to the principal and the trader is increasing with \( \sigma \) and \( \theta \), and the expected payoff to the liquidity traders is decreasing with \( \sigma \) and \( \theta \); (ii) the expected net payoff to the trader is increasing with \( \sigma \) and \( \kappa \), but decreasing with \( \overline{w} \) and \( \theta \); the expected payoff to the principal is decreasing with \( \kappa \), but increasing with \( \overline{w} \) and \( \theta \).

PROOF: See Appendix. ■

The first result is the same as Albert Kyle (1985), where a privately informed trader always benefits from a higher \( \sigma \) (a noisier liquidity order), while a liquidity trader is worse off with a higher \( \sigma \). The trader can benefit from a higher \( \sigma \) because the monitoring role of the price is less effective with a less informative price. When \( \sigma \) is smaller, the principal can offer a contract that is more sensitive to the price and thus reduce the payoff to his trader. However, how the principal’s expected payoff changes with \( \sigma \) is ambiguous.

With regard to \( \theta \), a higher precision in information production allows the information producers to take greater advantage of the liquidity traders, thus benefiting the principal. However, the trader is worse off with a higher \( \theta \), because there is less leniency for mistakes that are less likely to occur.

The results with regard to \( \kappa \) and \( \overline{w} \) can be interpreted as follows. When the cost of information production, \( \kappa \), is higher, the principal has to pay the trader more. If \( \overline{w} \) increases, then the optimal contract with the original lower \( \overline{w} \) is still feasible, but the higher \( \overline{w} \) generates slack with respect to the IC conditions, that is, the contract can be improved upon. Thus, a higher \( \overline{w} \) results in a lower payoff to the trader. Intuitively, when \( \overline{w} \) is higher, the contract in the form of “\( \overline{w} \) or nothing” allows the principal to pay more when the market price is more extreme, thus generating a stronger incentive for the trader to produce information.

To summarize, if there is an informative, verifiable signal – namely, the interim security market price — in addition to the final value, then, perhaps not surprisingly, it is optimal to include it as part of the optimal contract. This is the essential logic of the mark-to-
market contract. The direct trader has no agency problem and always produces information and trades optimally. Although his information is private, his order is going to move the market price set by the rational market maker. The principal makes use of this behavior of the direct trader to monitor his agent-trader. He free rides on the information in the price that is due to the direct trader.

The existence of the direct investor guarantees that the market price is informative, at least to some extent. However, the idea that the interim market price is informative does not rule out the possibility that traders will trade in suboptimal ways, and that they may alter the information reflected by the market price. In case this happens, the principals need to incorporate it into the optimal contracts, and may even use contracts that are not marked-to-market. In the next section, we begin exploring these issues in the trading context with multiple principal-agent pairs.

III. Competition in Contracts with Multiple Principal-Investors

In the previous section, the optimal mark-to-market contract takes advantage of the interim price that is formed in response to the trading positions of all the market participants, some of whom produce information. Due to a positive correlation of the signals received by different agents, a hard-working trader is likely to receive a high signal when the interim price is pushed up by the buying orders from other informed investors, and a low signal when the interim price is pushed down by the selling orders from other informed investors. By examining this consistency, the principal makes sure that the trader’s good performance is the result of hard-work, rather than the outcome of good luck.

If the informativeness of the interim market price can act as a monitoring device, then we have to address the question: where does the information come from? The assumption of the existence of a direct investor provides a trivial answer: it comes from the direct investor who trades for himself and is not subject to the moral hazard problem. What if the market is mainly dominated by delegated traders? In this case, the information has to come from other delegated traders. In other words, the information injected into the market by some traders is used to monitor other traders and vice versa. If traders understand the mutual monitoring feature of the interim price, then they can act strategically to jointly undo the monitoring effects of the price.

**LEMMA 1:** Suppose there are two delegated traders (each hired by a principal) who each receive the optimal mark-to-market contract in Proposition 2. Then if one trader shirks and buys (sells) \( x \) shares, the best response of the other trader is to follow. Joint shirking gives traders higher payoffs than what they receive by not shirking.

**PROOF:** See Appendix. ■

Roughly, when there are multiple delegated traders, they have an incentive to jointly shirk and trade in the same direction, moving the price, if they can find a way to
coordinate their trading. By so doing, they can “manipulate” the market such that the interim price is likely to move in line with their trading positions, enabling them to take advantage of the mark-to-market contract. Of course, if principal-investors anticipate this, then the optimal contract should reflect the possibility of this joint moral hazard problem. In the next two subsections, we formally study the equilibrium in which the traders coordinate to shirk and trade in the same direction due to the externality effect of a mark-to-market contract.

A. Pure Strategy Manipulation Equilibrium

To proceed, let us assume that there are two delegated traders in the market, and we introduce an irrelevant noise signal, $r$, which is, with probability $q$, observed by the traders before they make their effort to acquire information. The signal may be thought of as a news story, a rumor, a technical trading signal, a superstitious event, and so on. Traders regularly communicate, these days by Bloomberg mail and e-mail, and previously by phone. The social structure of markets, including the culture and communication, is essentially subsumed by our assumption of an irrelevant signal, but that background makes the idea quite plausible. See, for example, Mitchel Y. Abolafia (1996) and Karin Knorr-Cetina and Urs Bruegger (2002). So, $q$ is the likelihood that traders can, in some way, coordinate. The signal, $r$, is a reduced form for whatever method is used to coordinate. Note that we do not assume that traders can always coordinate (that is, $q = 1$); instead, we only consider the possibility of coordination and study how the equilibrium changes as the propensity for coordination changes. The optimal contract characterized in Proposition 2 is the solution for the special case where $q = 0$.

The noise signal is independent of the fundamentals: the relevant signal $s$ and the liquidation value $v$. It should be ignored. Further we assume that the irrelevant signal has two values: 0 or 1, and they are observed with equal probability. Upon observing the irrelevant signal, the traders both buy $x$ shares if the signal is 1 or both sell $x$ shares if the signal is 0, without acquiring information.

In this subsection, we will discuss the equilibrium in which only mark-to-market contracts are offered. We call the type of equilibrium Pure Strategy Manipulation Equilibrium, which we formally define below:

---

9 As we will show, the result in Lemma 1 still holds when the market price and the optimal contract reflect the possibility of joint shirking.

10 We take out the direct investor from the model mainly for two reasons: (i) With only two informed traders in the market, it is easy to compare the results in this section with those in the last section; (ii) This emphasizes the idea that the agent-traders can coordinate to shirk and herd in trading only when they constitute a substantial fraction of market participants and their trades could possibly move the market.

11 This equal probability assumption is purely for simplification, and our analysis below can be generalized to any probability specification between (including) zero and one.

12 Of course, the traders can buy (or sell) on both signal values, and that is equivalent to assuming that the probability of $r = 1$ (or 0) is equal to one.
DEFINITION 1 (A Pure Strategy Manipulation Equilibrium): A Pure Strategy Manipulation Equilibrium is a Nash equilibrium in which the principals offer mark-to-market contracts, the traders produce information and trade truthfully only when they do not observe the irrelevant signal. Upon observing the irrelevant signal, the traders both shirk and buy x shares if the signal is 1, and both sell x shares if the signal is 0.

In the above definition, the agent-traders coordinate on the irrelevant signal, $r$, to manipulate the market price. We will call the probability of this noise signal arriving, $q$, the “propensity of shirking”.

In a Pure Strategy Manipulation Equilibrium, the market maker cannot observe $r$, but rationally anticipates the traders’ actions and sets the price according to the aggregate order flow. The pricing function is:

$$p(q, z) = \frac{(1-q)[\phi^+(z)p_H + \phi^-(z)p_L]}{\phi^+(z)+\phi^-(z)} + \frac{1}{2} q(p_H+p_L). \quad (25)$$

Given the equilibrium pricing function $p(q, z)$, and with $A_{iq}^z$ defined similarly to (17), the optimal mark-to-market contract $w^M(p, \lambda, \nu)$, which allows for coordinated price manipulation by agent-traders upon observing the noise signal, must satisfy the following incentive constraints in addition to (IC.1) & (IC.2) in (18)-(19).

Without information, a trader is willing to buy/sell when the other trader is doing so:

$$(IC.M1) \quad \begin{align*}
\frac{1}{2}(A_{bh}^+ + A_{bl}^+) &\geq \frac{1}{2}(A_{sh}^0 + A_{sl}^0) \\
\frac{1}{2}(A_{sh}^- + A_{sl}^-) &\geq \frac{1}{2}(A_{bh}^0 + A_{bl}^0).
\end{align*} \quad (26)$$

Upon observing the irrelevant noise signal, a trader has an incentive to shirk and buy/sell (instead of producing information and trading truthfully) if the other trader is doing that:

$$(IC.M2) \quad \begin{align*}
\frac{1}{2}(A_{bh}^+ + A_{bl}^+) + \kappa &\geq \frac{1}{2} \left[ \theta A_{bh}^0 + (1-\theta)A_{bl}^+ + (1-\theta)A_{sh}^0 + \theta A_{sl}^0 \right] \\
\frac{1}{2}(A_{sh}^- + A_{sl}^-) + \kappa &\geq \frac{1}{2} \left[ \theta A_{bh}^0 + (1-\theta)A_{bl}^+ + (1-\theta)A_{sh}^0 + \theta A_{sl}^0 \right].
\end{align*} \quad (27)$$

If an incentive-compatible mark-to-market contract is offered to both agent-traders, the expected payoff to a trader is:
and the optimal contracting problem can be written as:

$$\min_{w \in W} A^M \quad \text{subject to the IC conditions in (18) - (19) & (26) - (27).} \quad (29)$$

We solve the optimal contracting problem (29) by first solving a weakened problem with only the IC condition (IC.2) in (19). Then we show that the optimal solution for the weakened problem satisfies (IC.1), (IC.M1), and (IC.M2).

By dropping (IC.1), (IC.M1), and (IC.M2) from the full programming problem in (29), we can write an alternative simplified programming problem:

$$\min_{w \in W} A^M \quad \text{subject to the IC conditions in (19).} \quad (30)$$

This problem is similar to the one in (21), and can be characterized in a similar way. First, analogously to Proposition 2, we can show that the constraints in (19) are binding for the optimal solution to (30), and the solution is characterized by four cutoff values \(z^* = \{z_{bH}^*, z_{bL}^*, z_{sH}^*, z_{sL}^*\}\). Define:

\[
\begin{align*}
\mu_1 &= \frac{1}{1-\theta} \frac{[(1-q)(1-\theta) + \frac{1}{2} q] \phi^+(z_{bL}^*) \phi^-(z_{sL}^*) + [(1-q)\theta + \frac{1}{2} q] \phi^0(z_{bL}^*) \phi^-(z_{sL}^*)}{\phi^0(z_{bL}^*) \phi^0(z_{sL}^*) - \phi^+(z_{bL}^*) \phi^-(z_{sL}^*)} \\
\mu_2 &= \frac{1}{1-\theta} \frac{[(1-q)(1-\theta) + \frac{1}{2} q] \phi^+(z_{bH}^*) \phi^-(z_{sH}^*) + [(1-q)\theta + \frac{1}{2} q] \phi^0(z_{bH}^*) \phi^0(z_{sH}^*)}{\phi^0(z_{bH}^*) \phi^0(z_{sH}^*) - \phi^+(z_{bH}^*) \phi^-(z_{sH}^*)}.
\end{align*}
\]  

(31)

In the following proposition, we first characterize the solution to (30), and then show that it is also the solution to (29).

**PROPOSITION 5 (Characterization of the Optimal Contract):** In a Pure Strategy Manipulation Equilibrium, (i) the optimal contract \(w^*(p, \lambda, v)\) takes the following form: when \(\lambda = b\), there exists \(z_{bH}^*\) such that, for any \(p < p(q, z_{bH}^*), w^*(p, b, v_q) = 0\), and for any \(p \geq p(q, z_{bH}^*), w^*(p, b, v_q) = \overline{w}\); when \(\lambda = s\), there exists \(z_{sH}^*\) such that, for any \(p > p(q, z_{sH}^*), w^*(p, s, v_s) = 0\), and for any \(p \leq p(q, z_{sH}^*), w^*(p, s, v_s) = \overline{w}\). Moreover, (ii) \(z_{jH}^*\) satisfy the following conditions:
\[ z_{bh}^* - z_{bl}^* = \frac{\sigma^2}{2x} \left[ \ln \left( \frac{1 - \theta}{\theta} \right) + \ln \left( \frac{2(1 - \theta)[(1 - q) + \mu_1] + q}{2\theta(1 - q) + \mu_1 + q} \right) \right] \]  

(32)

\[ z_{sH}^* - z_{sl}^* = \frac{\sigma^2}{2x} \left[ \ln \left( \frac{1 - \theta}{\theta} \right) + \ln \left( \frac{2(1 - \theta)[(1 - q) + \mu_2] + q}{2\theta(1 - q) + \mu_2 + q} \right) \right], \]

(33)

\[ \frac{1}{2} \left[ \theta \left\{ [1 - \Phi^+(z_{bH}^*)] + (1 - \theta)[1 - \Phi^+(z_{bl}^*)] \right\} \right] + \frac{1}{2} \left[ \theta \Phi^0(z_{sH}^*) + (1 - \theta)[1 - \Phi^0(z_{sl}^*)] \right] + \kappa \]

\[ \frac{1}{2} \left[ (1 - \theta) \Phi^-(z_{sH}^*) + \theta \Phi^-(z_{sl}^*) \right] = \frac{1}{2} \left[ \Phi^0(z_{bH}^*) + \theta \Phi^0(z_{bl}^*) + \kappa \right], \]  

and

\[ z_{bh}^* - z_{sH}^* = z_{bl}^* - z_{sl}^* > 2x. \]  

(34)

PROOF: See Appendix. ■

As before, the four equations in (32) and (33) allow for the determination of the four \( z \)-cutoff points. Immediately, we can check that when \( q = 0 \), the above proposition is reduced to Proposition 2. Proposition 5 also implies that \( A_{bH}^0 < A_{bL}^* \) and \( A_{sH}^0 < A_{sL}^* \), which says that under the optimal contract, when the trader buys/sells the stock, he will be better off if the other trader is doing the same. This generates the externality effect, which is at the root of the existence of an equilibrium with manipulation.

As in the last section, in order to demonstrate that the optimal contract characterized above can be sustained in a Nash equilibrium, we need to check that the payoff to the principal is higher than his alternative payoff from offering a standard contract.

In a Pure Strategy Manipulation Equilibrium, the expected payoff of the investment under informed trading is:

\[ V^M = \frac{1}{2} (1 - q) x \left\{ \int_{-\infty}^{\infty} [p_H - p(q, z)] \Phi^+(z) \, dz + \int_{-\infty}^{\infty} [p(q, z) - p_L] \Phi^-(z) \, dz \right\} \]

\[ + \frac{1}{2} q x \left\{ \int_{-\infty}^{\infty} [p_H + p_L - 2p(q, z)] \Phi^+(z) \, dz + \int_{-\infty}^{\infty} [p(q, z) - P_H + P_L] \Phi^-(z) \, dz \right\}. \]

(35)

Define the principal’s payoff from using a mark-to-market contract as \( V^M_p = V^M - A^M \), with \( V^M \) and \( A^M \) defined in (35) and (28), respectively. Given all the other players’ strategies (including the market maker’s pricing strategy), if one principal deviates and offers a standard contract while the other principal is offering a mark-to-market contract, the expected joint payoff to the principal and the trader can be written as:

\[ V^S = \frac{1}{2} x \left\{ \int_{-\infty}^{\infty} [p_H - p(q, z)] \Phi^+(z) \, dz + \int_{-\infty}^{\infty} [p(q, z) - p_L] \Phi^-(z) \, dz \right\}. \]

(36)

The principal’s expected payoff is \( V^S_p = V^S - A^S \), with \( A^S \) defined in (14). We need to
have $V_p^M \geq V_p^S$ for a principal not to deviate to a standard contract. Define the difference as $\Delta V_P = V_p^M - V_p^S$. A Pure Strategy Manipulation Equilibrium exists when $\Delta V_P > 0$.

PROPOSITION 6 (Existence): We have (i) $V^S = V^M + \frac{1}{2} qx(p_H - p_L) > V^M$ and $A^S > A^M$; (ii) $V_p^M$ is decreasing with $q$, $V_p^S$ is increasing with $q$, and $\Delta V_P$ is decreasing $q$; and (iii) There exists a $\bar{q} \in (0,1)$ such that a Pure Strategy Manipulation Equilibrium exists if and only if $0 \leq q \leq \bar{q}$.

PROOF: See Appendix.■

While $V^S > V^M$ reflects the benefit of using a standard contract, $A^S > A^M$ reflects the cost of using a standard contract. On the one hand, a standard contract precludes the possibility of market manipulation and provides the agent-trader with an incentive to acquire information. On the other hand, such a contract results in more costly monitoring due to its failure to fully utilize the information in the market price to monitor the agent-trader. Proposition 6 says that, with a higher propensity of shirking, a principal is worse off when a mark-to-market contract is used, and he has an incentive to deviate to a standard contract. Obviously, a mark-to-market contract is used only when $q$ is not too large, when the cost of a standard contract dominates its benefit.

A small $q$ has three roles: (i) It guarantees that the space of incentive-compatible contracts is not empty when the price is less informative due to coordinated manipulation; (ii) It ensures that the principal has a sufficient incentive to hire an agent-trader even though the agents in the economy coordinate to manipulate the price with some probability; (iii) It ensures that the benefit of using a standard contract in comparison with a mark-to-market contract is not large enough to offset its cost.

In a Pure Strategy Manipulation Equilibrium, no matter whether traders trade based on information or manipulate the price, they trade in the same direction. Their trading positions are always consistent with the interim market price. This is consistent with the empirical evidence found by Mark Grinblatt, Sheridan Titman, and Russ Wermers (1995) in many aspects. First, fund managers herd and the herding behavior is not necessarily always based on the information. Second, fund managers use momentum strategies and do not bet against the market. Finally even if fund managers do not always trade on information, their overall performance is still positive because they do trade on information when there is no chance to coordinate.

Proposition 6 does not tell us what happens when the propensity of shirking, $q$, is greater than $\bar{q}$. At least one principal will deviate to a standard contract as $q$ increases. However, there cannot be a symmetric pure strategy equilibrium with both principals using standard contracts no matter how large $q$ is. When one principal offers a standard contract, his trader will gather information and trade truthfully, but then the other principal will be better off offering a mark-to-market contract, as there is no risk of price manipulation. In the next subsection, we analyze a mixed strategy equilibrium, in which the principals mix
between a mark-to-market contract and a standard contract.

B. A Mixed Strategy Manipulation Equilibrium

When \( q \) gets large, there are two effects: First, it is possible that the gain from using the market price as a monitoring device is cancelled out by the cost of hiring a trader, due to the traders’ joint manipulation of the market price. Second, maybe more interestingly, the benefit of a standard contract becomes larger than its cost.

Because of the second effect, it is not a pure strategy equilibrium for both principals to adopt mark-to-market contracts. In this case, principal-investors may choose to hire agent-traders with a standard contract. As we have already discussed, a standard contract cannot sustain in pure strategy equilibria, so a mixed strategy equilibrium may exist. Intuitively, the benefit of a standard contract grows as the propensity of shirking, \( q \), increases, and when \( q \) is too large, a Pure Strategy Manipulation Equilibrium breaks down. However, mixing between a mark-to-market contract and a standard contract might mitigate the second effect because the benefit of a standard contract decreases as the probability of the other principal using a standard contract increases, until a principal is indifferent between the two types of contract.

More specifically, assume that traders can observe each other’s contracts, thus they can coordinate to shirk when they both receive a mark-to-market contract. If at least one trader receives a standard contract, they have no choice but to produce information and trade truthfully because the standard contract does not depend on the interim price. However, when both traders receive a mark-to-market contract, they can coordinate on the irrelevant signal to shirk. Therefore, the principals can mix between a standard contract and a mark-to-market contract to reduce the overall propensity of manipulation.

DEFINITION 2 (A Mixed Strategy Manipulation Equilibrium): A Mixed Strategy Manipulation Equilibrium is a Nash equilibrium in which principals and traders adopt the following strategies:

(i) Each principal independently offers a mark-to-market contract with probability \( m \) and a standard contract with probability \( 1 - m \);

(ii) If both traders receive mark-to-market contracts, then they shirk upon observing an irrelevant noise signal \( r \), with probability \( q \); otherwise they collect information and trade truthfully.

With the equilibrium strategies described above, shirking will occur with probability \( \hat{q} = m^2 q \), when both traders receive mark-to-market contracts and the irrelevant signal appears. To differentiate \( \hat{q} \) from \( q \), we call \( \hat{q} \) the “propensity of manipulation.” A Pure Strategy Manipulation Equilibrium is a special case of a Mixed Strategy Manipulation Equilibrium with \( m = 1 \) and only exists when \( 0 \leq q \leq \hat{q} \), where \( \hat{q} \) and \( q \) are exactly the same.

In a Mixed Strategy Manipulation Equilibrium, the market maker will set the stock
pricing function as:

\[ p(q, m, z) = \frac{(1 - \hat{q})[\phi^+(z) p_H + \phi^-(z) p_L]}{\phi^+(z) + \phi^-(z)} + \frac{1}{2} \hat{q}(p_H + p_L). \]  

We now discuss how \( m \) is determined in a Mixed Strategy Manipulation Equilibrium described above. When a mark-to-market contract is offered, the expected joint payoff to the principal and the trader can be written as:

\[
V^M(m) = \frac{1}{2} (1 - mq)x \left[ \int_{-\infty}^{\infty} [p_H - p(q, m, z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p(q, m, z) - p_L]\phi^-(z)dz \right] \\
+ \frac{1}{2} mqx \left[ \int_{-\infty}^{\infty} \frac{p_H + p_L}{2} - p(q, m, z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p(q, m, z) - \frac{p_H + p_L}{2}]\phi^-(z)dz \right] \\
= \frac{1}{2} x \left[ \int_{-\infty}^{\infty} [p_H - p(q, m, z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p(q, m, z) - p_L]\phi^-(z)dz \right] - \frac{1}{2} mq(x(p_H - p_L)).
\]  

The expected wage payment to the trader can be written as:

\[
A^M(m) = \frac{1}{2} (1 - mq)\left[ \theta A^+_{bh} + (1 - \theta)A^+_{bl} + (1 - \theta)A^-_{sl} + \theta A^-_{sl} \right] \\
+ \frac{1}{4} mq \left( A^+_{bh} + A^+_{bl} + A^-_{sh} + A^-_{sl} \right),
\]  

and the expected payoff to the principal is \( V^M_p(m) = V^M(m) - A^M(m) \).

Similar to a Pure Strategy Manipulation Equilibrium, the optimal mark-to-market contract solves the following problem:

\[
\min_{w \in \mathcal{W}} A^M(m) \text{ subject to the IC conditions in (18) - (19) & (26) - (27).}
\]  

The incentive constraints remain the same as in a Pure Strategy Manipulation Equilibrium.

**PROPOSITION 7 (Existence):** When \( \bar{q} \leq q \leq \bar{q} \), with \( \bar{q} \) being the maximum value of \( q \) such that a Pure Strategy Manipulation Equilibrium exists, there exists a unique Mixed Strategy Manipulation Equilibrium in which a principal offers a mark-to-market contract with probability \( m^*(q) \) and a standard contract with probability \( 1 - m^*(q) \) with \( m^*(q) \in (0,1) \). In addition, \( m^*(q) \) is decreasing in \( q \).

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13 As always, we assume that the market maker cannot observe the contracts received by the traders and does not know when manipulation exactly occurs. If the market maker can observe the contracts, then he can set the price contingent on the principals’ contract choices at the interim date. Our results would still hold, but the analysis would be more complicated.
The principals’ mixed strategies introduce noise and have an economic meaning in our context. The mixed strategy mitigates the coordinated shirking and price manipulation rooted in the contract externality embedded in mark-to-market contracts. For a large $q$, the agency problem is so large that no agent-traders will be hired if they shirk upon noise signals. However, mixing between a standard contract and a mark-to-market contract reduces the propensity of manipulation from $q$ to $(m^*)^2q$. In other words, the principals’ mixed strategy reduces the traders’ manipulation—kind of fighting noise with noise. This speaks directly to the fact that the existence of a Mixed Strategy Manipulation Equilibrium does not require a small $q$, which is in contrast to the result in Proposition 6, where we do need a small $q$ to guarantee the existence of a Pure Strategy Manipulation Equilibrium.

In the mixed strategy equilibrium of Proposition 7, the principal sometimes bases his trader’s compensation on marking-to-market and sometimes not. At his discretion, the principal decides on the contract form, and consequently on the tightness of risk management. If the chance of manipulation is very high, then the interim price is less informative and cannot be used to tighten the monitoring of traders. The variation in compensation contracts we observe in the real world might be explained by how information is provided and exploited by the principals in the mixed strategy equilibrium. If some principals provide standard contracts, others have incentives to exploit the information in the interim price using mark-to-market contracts. On the other hand, if too many principals use mark-to-market contracts, manipulation makes interim price less useful and some principals have incentives to deviate to standard contracts.

Finally, notice that the mixed strategy means that the informativeness of the market price varies through time as a function of the principals’ contract choice.

C. The Payoff Distribution and Market Efficiency

We now discuss how the payoffs to the market participants change with $q$. In the proof for Proposition 7, we show that $\partial m^*/\partial q = -m^*/q$, which implies that $\partial q/\partial q = -(m^*)^2 < 0$. Therefore, as $q$ increases, the equilibrium propensity of manipulation, $q$, first increases, but then decreases after $q > \bar{q}$. The next proposition demonstrates how $q$ affects the payoffs to the market participants.

PROPOSITION 8 (Comparative Statics): (i) Agent-traders always benefit from a higher $q$, while the expected payoff to principal-investors always decreases in $q$. (ii) In a Pure Strategy Manipulation Equilibrium, the expected payoff to the liquidity trader is increasing in $q$, while in a Mixed Strategy Manipulation Equilibrium, that payoff is decreasing in $q$.

PROOF: See Appendix. ■
The externality of the mark-to-market contract, indexed by \( \hat{q} \), generates incentives for traders to coordinate on irrelevant signals, and this benefits the agent-traders and the liquidity traders at a cost to the principal-investors. This effect only occurs when \( q \leq \bar{q} \). When \( q > \bar{q} \), the manipulation problem becomes too severe, and the principals use the costly standard non-mark-to-market contracts to prevent the traders from shirking. As \( q \) increases, it is more likely that a standard contract will be used. Thus the principals suffer and the traders benefit from a higher \( q \) despite the fact that the realized propensity of manipulation \( \hat{q} \) is lower, as the second effect dominates the first effect. At the same time, the liquidity traders suffer from a lower realized propensity of manipulation, \( \hat{q} \), because there is more informed trading.

The most important implication of the model concerns the informativeness of the price. Because of traders’ coordinated manipulation and principals’ contract randomization, additional noise is added into the asset price and price informativeness is reduced.

**PROPOSITION 9 (Informativeness of Market Price):** Let \( \hat{q} \) be the propensity of market manipulation in a Manipulation Equilibrium. Price volatility is decreasing in \( \hat{q} \). Also, when \( z > 0 \) (\( z < 0 \)), the market price is increasing (decreasing) in \( \hat{q} \).

**PROOF:** See Appendix. ■

Kyle (1985) defines market depth as the order flow required to move prices by one unit. Proposition 9 says that in a Manipulation Equilibrium the market maker does not move prices by as much in response to a given order flow because he recognizes that sometimes the trading by the agent-traders is not based on fundamental information. Traders sometimes coordinate on irrelevant signals to shirk, in which case no information is incorporated into price that moves solely on non-fundamental shocks. This explains why at times big surges and drops in the market cannot be supported by fundamentals. The principals’ contract randomization serves as a mechanism to mitigate uninformed price moves and improve price informativeness.

**D. Discussion**

In the above analysis we assumed away the direct investor. What if there are direct investors in the market? Our analysis in Sections II and III can be viewed as two extreme cases: one in which the market is dominated by large direct investors who are not subject to the agency problem; the other in which the market is dominated by large delegated traders who have incentives to engage in moral hazard. In the first case, large direct investors always inject information into the market, principals can free ride on this information and provide mark-to-market contracts. The information provided by the direct investors provides a clear signal to disrupt the coordinated manipulation by delegated traders. In the second case, the direct investors are not big enough, and the information they inject into the market is easily buried in the noise created by the agent-traders’ manipulation of the market.
IV. Conclusion

How does information come to be reflected in security prices? Grossman and Stiglitz (1980) addressed this question by introducing the notion of “noise,” which allows privately informed traders to profit from trading and, in this way, prices reflected their private information, at least partially. The “noise” provides camouflage so that informed traders can trade profitably. In our setting, principals hire traders and attempt to induce them to behave as in Grossman and Stiglitz (1980) by linking compensation partly to the security prices. That is, the principals try to use the information in market prices to monitor their agents. Indeed, if prices can be relied upon to be informative, such mark-to-market contracts are optimal.

Mark-to-market contracts in our model are consistent with risk management policies imposed on traders in the real world. By checking the alignment between traders’ behavior and the market, financial institutions want to make sure that traders make investments based on information and prevent them from engaging in risky gambles. We show that mark-to-market contracts may induce market manipulation when security markets are dominated by principal-agent pairs. Each principal-investor attempts to control his agent-trader by relying on the informativeness of the price, where the informativeness of the price depends on the behavior of other principals’ agents. Traders realize that if there is a way to coordinate their trades: they can ignore fundamentals (which requires a costly effort to discover) and trade in the same direction, manipulating their compensation by influencing the security price. Although trade is based on a mixture of both fundamental and non-fundamental information, the overall performance is still positive. These results are consistent with empirical evidence on mutual fund herding, momentum trading, and performance (see, e.g., Grinblatt, Titman, and Wermers (1995)).

To mitigate the agents’ manipulation, principals follow mixed strategies in contracts, making it harder for the agent-traders to manipulate their payoffs. This equilibrium contract randomization mitigates the contract externalities because it acts to prevent the coordination to shirk among the agent-traders. When it is easier for traders to coordinate to manipulate market price, the interim price is less useful for monitoring traders and some principals will exclude interim price information from their compensation schemes. This might be an explanation for the variation of contracts and risk management measures we observe in the real world and subject to future research.
References


Appendix

PROOF OF PROPOSITION 1: Solving the binding constraints in (IC.S2) gives us:
\[ w_{bh} - w_{sl} = \frac{2\kappa}{2\theta - 1}, \quad \text{and} \quad w_{sl} - w_{bl} = \frac{2\kappa}{2\theta - 1}. \]
The results are immediate. ■

PROOF OF PROPOSITION 2: We will only show the proof for (i); the proof for part (ii) follows a similar argument. We prove (i) by contradiction. Without loss of generality, assume that there exist \( z_1 \leq z_2 - \delta \) for some \( \delta > 0 \), such that, with some \( \varepsilon > 0 \), \( w^*(p(z), b, v_\eta) \geq \varepsilon \) for any \( z \in [z_1 - \delta/2, z_1 + \delta/2] \), and \( w^*(p(z), b, v_\eta) \leq -\phi^+(z_1)/\phi^+(z_2) \) for any \( z \in [z_2 - \delta/2, z_2 + \delta/2] \). Now construct a new wage schedule \( \hat{w}^*(p(z), b, v_\eta) \) as follows:
\[
\hat{w}^*(p(z), b, v_\eta) = \begin{cases} 
  w^*(p(z), b, v_\eta) - \varepsilon & \text{if } z \in [z_1 - \delta/2, z_1 + \delta/2] \\
  w^*(p(z), b, v_\eta) + \phi^+(z_1)/\phi^+(z_2) & \text{if } z \in [z_2 - \delta/2, z_2 + \delta/2] \\
  w^*(p(z), b, v_\eta) & \text{otherwise.}
\end{cases}
\]

With the constructed new wage schedule \( \hat{w}^*(p(z), b, v_\eta) \), the resulting \( \hat{A}_{bH}^* \) remains approximately the same, while the resulting \( \hat{A}_{bH}^* \) will be strictly smaller since \( \phi^0(z_1)/\phi^0(z_2) > \phi^+(z_1)/\phi^+(z_2) \). Therefore, the second inequality of the IC conditions in (19) becomes a strict inequality, which means that the contract can be strictly improved, a contradiction.

For the second part of the results, we write out the Lagrangian for the optimization problem with non-positive multipliers \( \mu_1 \) and \( \mu_2 \), and get the first order conditions:
\[
\theta \phi^+(z_{bh}^*) + \mu_1 \theta \phi^+(z_{bh}^*) - \mu_2 (1 - \theta) \phi^0(z_{bh}^*) = 0 \]
\[
(1 - \theta) \phi^0(z_{sl}^*) + \mu_1 (1 - \theta) \phi^+(z_{sl}^*) - \mu_2 \theta \phi^0(z_{sl}^*) = 0 \]
\[
(1 - \theta) \phi^-(z_{sl}^*) + \mu_1 (1 - \theta) \phi^-(z_{sl}^*) - \mu_2 (1 - \theta) \phi^0(z_{sl}^*) = 0 \]
\[
\theta \phi^-(z_{sl}^*) + \mu_2 \theta \phi^+(z_{sl}^*) - \mu_1 (1 - \theta) \phi^0(z_{sl}^*) = 0.
\]

Eliminating \( \mu_1 \) and \( \mu_2 \), we get:
\[
\frac{\phi^+(z_{bh}^*) - \phi^+(z_{bl}^*)}{\phi^0(z_{bh}^*)} = \frac{\phi^0(z_{sl}^*) - \phi^0(z_{bh}^*)}{\phi^0(z_{bl}^*)} \quad \text{and} \quad \frac{\phi^+(z_{sl}^*) - \phi^+(z_{bl}^*)}{\phi^-(z_{sl}^*)} = \frac{\phi^0(z_{bh}^*) - \phi^0(z_{sl}^*)}{\phi^-(z_{bh}^*)},
\]
which implies:
\[
z_{bh}^* - z_{bl}^* = z_{sl}^* - z_{bh}^* = \frac{\sigma^2}{\epsilon} \ln \left( \frac{1 - \theta}{\theta} \right) < 0.
\]
The two equations in (23) are the binding constraints. The condition in (24) is from non-positive Lagrange multipliers. ■

PROOF OF PROPOSITION 3: First, existence. We construct a set of cutoff values \( z^S = \{z_{bh}^S, z_{bl}^S, z_{sl}^S, z_{sl}^S\} \) such that the corresponding contract in the form of the step function as described in Proposition 2 generates the same payoff as a standard contract
with (IC.2) unbinding. To improve \( z^* = \{ z^*_bH, z^*_bL, z^*_sH, z^*_sL \} \), we only need to increase \( z_{bH} \) or decrease \( z_{sL} \) to lower the payoff to the trader and increase the payoff to the principal.

Next, we prove uniqueness. Assume that there exist two optimal contracts characterized by \( z^* \) and \( \hat{z}^* \), and without loss of generality assume \( z^*_{bH} > \hat{z}^*_{bH} \). From equations in (22) and (23), we know that: \( z^*_{bL} > \hat{z}^*_{bL} \), \( z^*_{sH} < \hat{z}^*_{sH} \), and \( z^*_{sL} < \hat{z}^*_{sL} \). However, this implies that \( z^* \) yields a strictly smaller expected payoff to the trader as measured in (20), which is a contradiction.

**PROOF OF PROPOSITION 4**: Define:

\[
\begin{align*}
\beta_{11} &\equiv \theta \phi^+(z^*_{bH}) + (1 - \theta) \phi^+(z^*_{bL}) \\
\beta_{21} &\equiv (1 - \theta) \phi^-(\hat{z}^*_{sH}) + \theta \phi^-(\hat{z}^*_{sL}) \\
\beta_{12} &\equiv \theta \phi^0(z^*_{bH}) + (1 - \theta) \phi^0(z^*_{bL}) \\
\beta_{22} &\equiv (1 - \theta) \phi^0(\hat{z}^*_{sH}) + \theta \phi^0(\hat{z}^*_{sL})
\end{align*}
\]

We can show \( \beta_{11} \beta_{21} - \beta_{12} \beta_{22} > 0 \).

With \( V \) defined in (13), to prove part (i), we only need to show

\[
\frac{\partial}{\partial \sigma} \int_{-\infty}^{\infty} \phi^+(z) \phi^-(z) \, dz > 0.
\]

We have:

\[
\begin{align*}
\frac{\partial}{\partial \sigma} \int_{-\infty}^{\infty} \phi^+(z) \phi^-(z) \, dz &\equiv \frac{\partial}{\partial \sigma} \int_{-\infty}^{\infty} \exp(-\hat{z}^2/2) \, d\hat{z} \\
&= \frac{4x}{\sigma^2} \int_{-\infty}^{\infty} \exp\left[-\left(\frac{z+4x/\sigma}{\sigma}\right)^2/2\right] \, dz \\
&= \frac{4x}{\sigma^2} \int_{-\infty}^{\infty} \exp\left(-\frac{z^2}{2}\right) \, dz = 0
\end{align*}
\]

The last inequality comes from the fact that \( \exp[-(4x/\sigma)(z-2x/\sigma)]+1 \) is a positive function decreasing in \( z \).

For results about \( \theta \), tedious algebra leads us to:

\[
\frac{\partial V}{\partial \theta} = \int_{-\infty}^{\infty} \frac{\phi^+(z) \phi^-(z)}{\phi^+(z) + \phi^-(z)} (v_H - v_L) [2\phi^+(z) + 2\phi^-(z)] \, dz > 0.
\]

For (ii), with the trader’s expected payoff \( A \) defined in (20), we can show:

\[
\frac{\Delta A}{\Delta \kappa} = -\beta_{11} \bar{w} \frac{\Delta z^*_bH}{\Delta \kappa} + \beta_{21} \bar{w} \frac{\Delta z^*_sH}{\Delta \kappa} = \frac{\beta_{11} (\beta_{21} + \beta_{12}) + \beta_{21} (\beta_{11} + \beta_{22})}{\beta_{11} \beta_{21} + \beta_{12} \beta_{22}} > 1.
\]

Similarly, we can show \( \Delta A/\Delta \bar{w} < 0 \). In addition, because the sum of the trader’s payoff and the principal’s payoff does not depend on \( \kappa \) or \( \bar{w} \), we must have the principal’s
payoff decrease in $\kappa$ and increase in $w$.

To show $\Delta A / \Delta \theta < 0$, applying the Envelope Theorem to the optimization problem in (21), with $\mu_1 < 0$ and $\mu_2 < 0$ defined in the proof for Proposition 2, we have:

$$
\frac{\partial A}{\partial \theta} = (A^+_{bH} - A^+_{bl} + A^-_{sl} - A^-_{sh}) + \mu_1 (A^+_{bH} - A^+_{bl} + A^0_{sl} - A^0_{sh}) + \mu_2 (A^-_{sl} - A^-_{sh} + A^0_{bH} - A^0_{bl})
$$

$$
= (1 + \mu_1)(A^+_{bH} - A^+_{bl}) + (1 + \mu_2)(A^-_{sl} - A^-_{sh}) + \mu_1 (A^0_{sl} - A^0_{sh}) + \mu_2 (A^0_{bH} - A^0_{bl}).
$$

It is easy to verify that $1 + \mu_1 < 0$ and $1 + \mu_2 < 0$, and we conclude $\Delta A / \Delta \theta < 0$.

Finally, we prove that the trader’s payoff is increasing in $\sigma$ by construction. Given $\sigma$, pick an optimal contract $w(p(z), \lambda, v_q)$ characterized by the cutoff points $\{z_{bH}, z_{bl}, z_{sh}, z_{sl}\}$, satisfying the IC conditions in (19). For $\hat{\sigma} < \sigma$, we construct the new wage functions $\hat{w}(p(z), \hat{\lambda}, \hat{v}_q)$, characterized by the cutoff points $\{\hat{z}_{bH}, \hat{z}_{bl}, \hat{z}_{sh}, \hat{z}_{sl}\}$, as follows:

$$
\frac{\hat{z}_{bH} - 2x}{\hat{\sigma}} = \frac{z_{bH} - 2x}{\sigma}, \quad \frac{\hat{z}_{bl} - 2x}{\hat{\sigma}} = \frac{z_{bl} - 2x}{\sigma}, \quad \frac{\hat{z}_{sh} + 2x}{\hat{\sigma}} = \frac{z_{sh} + 2x}{\sigma}, \quad \frac{\hat{z}_{sl} + 2x}{\hat{\sigma}} = \frac{z_{sl} + 2x}{\sigma}.
$$

With the constructed $\hat{w}(p(z), \hat{\lambda}, \hat{v}_q)$, we can show that:

$$
\hat{A}^+_{bH} = A^+_{bH}, \hat{A}^+_{bl} = A^+_{bl}, \hat{A}^-_{sh} = A^-_{sh}, \text{ and } \hat{A}^-_{sl} = A^-_{sl}.
$$

Therefore, the left hand side values of the IC conditions in (19) remain the same. At the same time, we can show that:

$$
\hat{A}^0_{bH} < A^0_{bH}, \hat{A}^0_{bl} < A^0_{bl}, \hat{A}^0_{sh} < A^0_{sh}, \text{ and } \hat{A}^0_{sl} < A^0_{sl}.
$$

That is, the right hand side values of the IC conditions in (19) get smaller. Hence, the IC conditions become strict inequalities with our construction, and we can improve the contract.

PROOF OF LEMMA 1: By working hard and trading according to the acquired information, the traders’ expected payoff is:

$$
\left[\theta A^+_{bH} + (1-\theta)A^+_{bl} + (1-\theta)A^-_{sh} + (1-\theta)A^-_{sl}\right]/2.\text{ If they shirk jointly, they can both buy the stocks or both sell the stocks. If they both buy, their payoff is: }\left(A^+_{bH} + A^+_{bl}\right)/2 + \kappa.\text{ The difference between payoffs from joint shirking and both working is equal to:}
$$

$$
\frac{1}{2}[\theta A^+_{bH} + (1-\theta)A^+_{bl} - (1-\theta)A^-_{sh} - \theta A^-_{sl}] + \kappa
$$

$$
= \frac{1}{2}[\theta A^+_{bH} + (1-\theta)A^+_{bl} - (1-\theta)A^0_{bH} - \theta A^0_{bl}] > 0,
$$

where the first equality comes from the binding constraints in (IC.2) in (19). The case of selling can be proved similarly.

PROOF OF PROPOSITION 5: We first ignore constrains (IC.1), (IC.M1), and (IC.M2). With (IC.2) in (19), the proof for (i) and (ii) of the first part is the same as for Proposition 1. The second part follows from the Lagrangian conditions for the optimization problem with non-positive multiplier $\mu_1$ and $\mu_2$. 

29
Now, we show that the solutions above also satisfy (IC.1), (IC.M1), and (IC.M2). It is obvious that (IC.2) implies (IC.1), and we only need to check (IC.M1) and (IC.M2). First observe that, with (IC.2) binding, (IC.M1) and (IC.M2) are equivalent.

To prove (IC.2) implies (IC.M2), using $A_{x_q}^0 < A_{x_q}^-$ and $A_{b_q}^0 < A_{x_q}^-$, we have:

$$
\frac{1}{2} \left[ \theta A_{b_H}^+ + (1 - \theta) A_{b_L}^- + (1 - \theta) A_{x_H}^0 + \theta A_{x_L}^0 \right]
< \frac{1}{2} \left[ \theta A_{b_H}^+ + (1 - \theta) A_{b_L}^- + (1 - \theta) A_{x_H}^- + \theta A_{x_L}^- \right]
$$

$$
= \frac{1}{2} \left[ \theta A_{b_H}^+ + (1 - \theta) A_{b_L}^- + (1 - \theta) A_{b_H}^0 + \theta A_{b_L}^0 \right] + \kappa \quad \text{(IC.2)}
$$

and similarly

$$
\frac{1}{2} \left[ \theta A_{b_H}^0 + (1 - \theta) A_{b_L}^- + (1 - \theta) A_{x_H}^0 + \theta A_{x_L}^- \right] < \frac{1}{2} \left( A_{x_H}^- + A_{x_L}^- \right) + \kappa. \quad \blacksquare
$$

**PROOF OF PROPOSITION 6:** The proof of $V^S > V^M$ is trivial if we compare (35) and (36). We know that a standard contract also satisfies the incentive constraints (IC.2), but it is not the solution to the optimal contracting problem (30). Therefore, we know that a standard contract does not yield the minimum wage payment, that is: $A^S > A^M$. Some manipulation with $V^M$ gives us:

$$
V_p^M = (1 - q) x (p_H - p_L) \left[ \frac{\phi^+(z) \phi^-(z)}{\phi^+(z) + \phi^-(z)} dz - \frac{1}{2} (1 - q) \left[ \theta A_{b_H}^+ + (1 - \theta) A_{b_L}^- + (1 - \theta) A_{x_H}^- + \theta A_{x_L}^- \right] \right]
$$

$$
- \frac{1}{4} \left( A_{b_H}^+ + A_{b_L}^- + A_{x_H}^- + A_{x_L}^- \right)
$$

We have:

$$
\frac{\partial V_p^M}{\partial q} = -x (p_H - p_L) \left[ \frac{\phi^+(z) \phi^-(z)}{\phi^+(z) + \phi^-(z)} dz + \frac{1}{2} \left[ \theta A_{b_H}^+ + (1 - \theta) A_{b_L}^- + (1 - \theta) A_{x_H}^- + \theta A_{x_L}^- \right] \right]
$$

$$
- \frac{1}{4} \left( A_{b_H}^+ + A_{b_L}^- + A_{x_H}^- + A_{x_L}^- \right)
$$

Because:

$$
\frac{1}{2} \left[ \theta A_{b_H}^+ + (1 - \theta) A_{b_L}^- + (1 - \theta) A_{x_H}^0 + \theta A_{x_L}^0 \right] < \frac{1}{2} \left( A_{b_H}^+ + A_{b_L}^- \right) + \kappa,
$$

and

$$
\frac{1}{2} \left[ \theta A_{b_H}^0 + (1 - \theta) A_{b_L}^- + (1 - \theta) A_{x_H}^- + \theta A_{x_L}^- \right] < \frac{1}{2} \left( A_{x_H}^- + A_{x_L}^- \right) + \kappa,
$$

we have $\frac{\partial V_p^M}{\partial q} < -(V - \kappa) < 0$, where $V$ is defined in (13), and $V > \kappa$ from Assumption 2.

At the same time, because $A^S$ is independent of $q$, we have:
\[
\frac{\partial V^S_P}{\partial q} = \frac{\partial V^S}{\partial q} = \frac{1}{4} x (p_H - p_L) \left[ \left( \phi^+(z) - \phi^-(z) \right)^2 \right] dz > 0.
\]

Then it follows that
\[
\frac{\partial \Delta V_P}{\partial q} = \frac{\partial V^M_P}{\partial q} - \frac{\partial V^S_P}{\partial q} < 0.
\]

We observe that the optimization problem in (30) converges to the one in (21) as \( q \) goes to zero, thus we get the existence of an optimal incentive-compatible contract to (30) by continuity under Assumption 2. In addition, we have
\[
\frac{\partial \Delta V_P}{\partial q} = \frac{\partial V^M_P}{\partial q} - \frac{\partial V^S_P}{\partial q} < 0, \quad \Delta V_P > 0
\]
when \( q = 0 \), and \( \Delta V_P < 0 \) when \( q = 1 \). Therefore there must exist a unique \( \bar{q} \in (0,1) \) such that \( \Delta V_P \geq 0 \) if and only if \( 0 \leq q \leq \bar{q} \). Moreover, it is easy to see that under Assumption 2, at \( \bar{q} \), \( V^M_P = V^S_P > 0 \), which is also true for any \( q \leq \bar{q} \) as \( \frac{\partial V^M_P}{\partial q} < 0.14^\text{■} \)

PROOF OF PROPOSITION 7: When a standard contract is offered, the expected joint payoff to the principal and the trader can be written as:
\[
V^S(m) = \frac{1}{2} x \left[ [p_H - p(q, m, z)] \phi^+(z)dz + \int_{-\infty}^{\infty} [p(q, m, z) - p_L] \phi^-(z)dz \right]
\]
\[
= V^M(m) + \frac{1}{2} m q x (p_H - p_L) > V^M(m).
\]

The expected payoff to the trader is:
\[
A^S(m) = \frac{1}{2} [(\partial w_bhH + (1-\theta)w_{bL} + \partial w_{sL} + (1-\theta)w_{bL}],
\]
and the expected payoff to the principal is \( V^S_P(m) = V^S(m) - A^S(m) \).

Define the difference between \( V^M_P(m) \) and \( V^S_P(m) \) as \( \Delta V_P(m) = V^M_P(m) - V^S_P(m) \). We have
\[
\frac{\partial V^S_P}{\partial m} - \frac{\partial V^S}{\partial m} = \frac{\partial V^S}{\partial q} \frac{\partial q}{\partial m} = \frac{1}{4} x m q (p_H - p_L) \left[ \left( \phi^+(z) - \phi^-(z) \right)^2 \right] \phi^+(z) + \phi^-(z) dz > 0.
\]

At the same time, we have:
\[
\Delta V_P(m) = V^M_P(m) - V^S_P(m) = -\frac{1}{2} m q x (p_H - p_L) - [A^M(m) + A^S(m)].
\]

We know that \( \frac{\partial A^S(m)}{\partial m} = 0 \), and applying the Envelope Theorem to the optimization problem (40), we have:
\[
\frac{\partial A^M(m)}{\partial m} = -\frac{q}{2} \left[ \partial A^+_bH + (1-\theta)A^-_{bL} + (1-\theta)A^-_{sL} + \partial A^-_{bL} \right] + \frac{q}{4} (A^+_bH + A^+_bL + A^-_{sH} + A^-_{sL}), \quad \text{and}
\]

---

14 In the existence proof, we only consider the possible deviation to choosing a standard contract. There are other possible deviations. For example, we can consider a mark-to-market contract that excludes shirking by reversing the inequalities in (IC,M1) and (IC,M2), however, we can formally show that such deviation is not possible. Intuitively, any mark-to-market contract will compensate traders when their trading position is consistent with the market price, and such compensation schemes always encourage coordinated shirking.
\[
\frac{\partial \Delta V_p(m)}{\partial m} = \frac{q}{2} \left[ \theta A_{bl}^+ + (1 - \theta) A_{bl}^- + (1 - \theta) A_{sh}^- + \theta A_{sh}^+ \right] - \frac{q}{4} \left( A_{bl}^+ + A_{sh}^- + A_{sh}^+ + A_{bl}^- \right) - \frac{q}{2} x(p_H - p_L) < q \left[ \kappa - x(p_H - p_L) \right] \int_{-\infty}^{+\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} \, dz = -q(V - \kappa) < 0,
\]

where \( V \) is defined in (14), and \( V > \kappa \) under Assumption 2.

Under Assumption 2, we know from the proof of Proposition 6 that when \( q > \bar{q} \), \( V_p^M(m = 1) < V_p^S(m = 1) \). At the same time, we know \( V_p^M(m = 0) > V_p^S(m = 0) \) because \( V^S(m = 0) = V^M(m = 0) \) and \( A^S(m = 0) > A^M(m = 0) \). By continuity, we know that there exists \( m^* \in (0,1) \), such that \( V_p^M(m^*) = V_p^S(m^*) > 0 \) because \( V_p^S(m = 0) > 0 \) and \( V_p^S(m) \) is increasing with \( m \).

The uniqueness directly comes from \( \partial \Delta V_p(m) / \partial m < 0 \). Finally, we have:

\[
\frac{\partial m^*}{\partial q} = -\frac{\partial \Delta V_p(m) / \partial q}{\partial \Delta V_p(m) / \partial m^*} = -\frac{m^*}{q} < 0,
\]

where \( \partial \Delta V_p(m) / \partial q < 0 \), which can be proved in a way similar to prove \( \partial \Delta V_p(m) / \partial m < 0 \).

**PROOF OF PROPOSITION 8:** Let us first prove the results for pure strategy equilibrium. In a Pure Strategy Manipulation Equilibrium, the expected net payoff to the trader is: \( V_T^M = A^M - (1 - q)\kappa \) and we have:

\[
\frac{\partial V_T^M}{\partial q} = -\frac{1}{2} \left[ \theta A_{bl}^+ + (1 - \theta) A_{bl}^- + (1 - \theta) A_{sh}^- + \theta A_{sh}^+ \right] + \frac{1}{4} \left( A_{bl}^+ + A_{sh}^- + A_{sh}^+ + A_{bl}^- \right) + \kappa.
\]

From the proof of Proposition 6, we know the above expression has a strictly positive value, which implies that the trader’s expected net payoff is increasing with \( q \). At the same time, \( V_p^M = V^M - A^M \) is decreasing with \( q \). We also have:

\[
\frac{\partial V^M}{\partial q} = -x(p_H - p_L) \int_{-\infty}^{+\infty} \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} \, dz < 0,
\]

Since we have \( V_L^M + 2V^M = 0 \), where \( V_L^M \) is the liquidity trader’s payoff. Therefore, the liquidity trader’s expected payoff is increasing with \( q \).

In a Mixed Strategy Manipulation Equilibrium, the payoff to the principal, denoted as \( V_p^M(m) \), is the same as using a standard contract, which is decreasing in \( q \) or increasing in \( \hat{q} \) as \( \hat{q} / \hat{q} = \hat{(m^*)^2} / \hat{q} - (m^*)^2 < 0 \) using \( \hat{m}^* / \hat{q} = -m^* / q < 0 \) (shown in the proof of Proposition 7). To see this, we have:

\[
\frac{\partial V_p^M(m)}{\partial \hat{q}} = \frac{\partial V^S(m)}{\partial \hat{q}} - \frac{\partial A^S(m)}{\partial \hat{q}} = \frac{\partial V^S(m)}{\partial \hat{q}}.
\]
as $A^S(m)$ is independent of $q$ or $\hat{q}$. Moreover, similar to (37), we have:

$$V^S(m) = \frac{1}{2} x \left[ \int_{-\infty}^{\infty} [p_H - p(q, m, z)]\phi^+(z)dz + \int_{-\infty}^{\infty} [p(q, m, z) - p_L]\phi^-(z)dz \right]$$

and

$$\frac{\partial V^S(m)}{\partial \hat{q}} = \frac{1}{4} x(p_H - p_L) \int [\phi^+(z) - \phi^-(z)]^2 d\hat{q} > 0,$$

as similarly shown in Proposition 6. This implies $\frac{\partial V^M(m)}{\partial \hat{q}} > 0$ or $\frac{\partial V^M(m)}{\partial \hat{q}} < 0$.

Let us denote the liquidity trader’s payoff by $V^M_L(m)$. The same algebra as in a pure strategy equilibrium gives us:

$$\frac{\partial V^M_L(m)}{\partial \hat{q}} = 2x(p_H - p_L) \int \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} d\hat{q} > 0,$$

which implies:

$$\frac{\partial V^M_L(m)}{\partial q} = \frac{\partial V^M_L(m)}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial q} = -\left(2x(p_H - p_L) \int \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} d\hat{q} \right)(m^*)^2 < 0,$$

as $\frac{\partial \hat{q}}{\partial q} = \hat{q}(m^*) q / (m^*)^2 + 2m^* q (\hat{m} / \partial \hat{q}) = -(m^*)^2$, where we used $\hat{m} / \partial \hat{q} = -m^* / q$.

The joint net payoff to the principal-investor and the agent-trader is the joint trading payoff, $V^M(m)$, net of the information production, $(1 - \hat{q})\kappa$. Therefore, the net payoff to the trader can be written as:

$$V^M(m) = V^M_L(m) - (1 - \hat{q})\kappa - V^M_P(m).$$

We know in equilibrium $V^M_L(m) + 2V^M(m) = 0$, thus, using $\frac{\partial V^M_P(m)}{\partial \hat{q}} < 0$ and Assumption 2 we have:

$$\frac{\partial V^M_T(m)}{\partial \hat{q}} = \left(\frac{1}{2} \frac{\partial V^M_L(m)}{\partial \hat{q}} + \kappa\right)\frac{\partial \hat{q}}{\partial \hat{q}} - \frac{\partial V^M_P(m)}{\partial \hat{q}}$$

$$= \left(x(p_H - p_L) \int \frac{\phi^+(z)\phi^-(z)}{\phi^+(z) + \phi^-(z)} dz - \kappa \right)(m^*)^2 - \frac{\partial V^M_P(m)}{\partial \hat{q}} > 0.$$

\[
\text{PROOF OF PROPOSITION 9: The results follow from}
\]

$$E\left[ (p(m, q, z) - E[p(m, q, z)])^2 \right] = (1 - \hat{q})^2 \int_{-\infty}^{\infty} \frac{(p_H - p_L)^2[\phi^+(z) - \phi^-(z)]^2}{8[\phi^+(z) + \phi^-(z)]} dz$$

and

$$p(m, q, z) - p(z) = \frac{-(p_H - p_L)\hat{q}[(\phi^+(z) - \phi^-(z)]}{2[\phi^+(z) + \phi^-(z)]}.$$